YAKEEN

NEET 2.0 2027

PHYSICS | Lecture -05

Vectors

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Topics to be covered

1. Dot Product

Q $\vec{A} = 3\hat{i} + 4\hat{j} + 5\hat{k}$

$\vec{B} = 2\hat{i} - 3\hat{j} + 6\hat{k}$

$\vec{A} \cdot \vec{B} = 6 - 12 + 30$

$= 24$

Dot product

$\vec{A} \cdot \vec{B} = AB\cos\theta$

$\cos\theta = \frac{\vec{A} \cdot \vec{B}}{AB}$

→ Angle b/w $\vec{A}$ & $\vec{B}$.

Q $\vec{A} = 3\hat{\imath} + 4\hat{\jmath} + 5\hat{k}$

$\vec{B} = \hat{\imath} + \hat{\jmath} + \hat{k}$

find angle b/w $\vec{A}$ & $\vec{B}$

$|A| = A = \sqrt{3^2 + 4^2 + 5^2}$

$= 5\sqrt{2}$

Sol $\vec{A} \cdot \vec{B} = AB\cos\theta$

$\cos\theta = \frac{\vec{A} \cdot \vec{B}}{AB} = \frac{3+4+5}{5\sqrt{2} \times \sqrt{3}} = \frac{12}{5\sqrt{6}}$

$\theta = \cos^{-1}\left(\frac{12}{5\sqrt{6}}\right)$

Q find angle b/w $\vec{A}$ & $\vec{B}$

① $\vec{A} = 2\hat{\imath} + 3\hat{\jmath} - 4\hat{k}$
$\vec{B} = 3\hat{\imath} + 4\hat{\jmath} + 2\hat{k}$

② $\vec{A} = 3\hat{\imath} - 4\hat{\jmath} + 5\hat{k}$
$\vec{B} = 5\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$

③ $\vec{A} = \hat{\imath} + \hat{\jmath} + \hat{k}$
$\vec{B} = 3\hat{\imath} + 3\hat{\jmath} + 3\hat{k}$

④ $\vec{A} = \hat{\imath} + \hat{\jmath}$
$\vec{B} = \hat{\jmath} - \hat{k}$

* ⑤ $\vec{A} = \hat{\imath} + \hat{\jmath}$
$\vec{B} = -\hat{\jmath}$

Q find angle b/w $\vec{A}$ & $\vec{B}$

① $\vec{A} = 2\hat{\imath} + 3\hat{\jmath} - 4\hat{k}$

$\vec{B} = 3\hat{\imath} + 4\hat{\jmath} + 2\hat{k}$

$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{AB} = \frac{6+12-8}{\sqrt{2^2+3^2+4^2} \times \sqrt{3^2+4^2+2^2}} = \frac{10}{29}$$

Shortcut - Exam

$A \equiv (2, 3, -4)$

$B \equiv (3, 4, 2)$

$6 + 12 - 8 = \checkmark$

② $\vec{A} = 3\hat{\imath} - 4\hat{\jmath} + 5\hat{k}$

$\vec{B} = 5\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$

Sol $\cos\theta = \frac{\vec{A}\cdot\vec{B}}{AB} = \frac{15+12+20}{\sqrt{50}\sqrt{50}} = \frac{47}{50}$

Q find ang b/w $\vec{A}$ & $\vec{B}$

③ $\vec{A} = \hat{\imath} + \hat{\jmath} + \hat{k}$

$\vec{B} = 3\hat{\imath} + 3\hat{\jmath} + 3\hat{k}$

$3\vec{A} = \vec{B}$

$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{AB} = \frac{3+3+3}{\sqrt{3} \times 3\sqrt{3}} = \frac{9}{9} = 1$

$\theta = 0$

④ $\vec{A} = \hat{\imath} + \hat{\jmath} =$

$\vec{B} = \hat{\jmath} - \hat{k} =$

$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{AB} =$

* ⑤ $\vec{A} = \hat{\imath} + \hat{\jmath}$

$\vec{B} = -\hat{\jmath}$

Q find ang b/w $\vec{A}$ & $\vec{B}$

③ $\vec{A} = \hat{\imath} + \hat{\jmath} + \hat{k}$
$\vec{B} = 3\hat{\imath} + 3\hat{\jmath} + 3\hat{k}$
$\Rightarrow$ $(1, 1, 1)$, $(3, 3, 3)$

$3 + 3 + 3 = 9$

④ $\vec{A} = \hat{\imath} + \hat{\jmath} \equiv (1, 1, 0)$
$\vec{B} = \hat{\jmath} - \hat{k} \equiv (0, 1, -1)$

$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{AB} = \frac{1}{\sqrt{2}\cdot\sqrt{2}} = \frac{1}{2}$$

$\theta = 60^\circ$

⑤ $\vec{A} = \hat{\imath} + \hat{\jmath} \equiv (1, 1, 0)$
$\vec{B} = -\hat{\jmath} \equiv (0, -1, 0)$

$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{AB} = \frac{0 - 1 + 0}{\sqrt{2} \times (1)} = \frac{-1}{\sqrt{2}}$$

$\theta = 135^\circ$

* $\vec{A} = \hat{i}$
$\vec{B} = \hat{j}$

$\vec{A} \cdot \vec{B} = AB\cos\theta$ $(\theta = 90)$

$\vec{A} \cdot \vec{B} = 0$

$\hat{i} \cdot \hat{j} = 0$

* $\hat{i} \cdot \hat{j} = 0$	$\hat{i} \cdot \hat{i} = 1$
* $\hat{i} \cdot \hat{k} = 0$	$\hat{j} \cdot \hat{j} = 1$
* $\hat{j} \cdot \hat{k} = 0$	$\hat{k} \cdot \hat{k} = 1$

$\hat{j} \cdot \hat{j} = 1 \times 1 \times \cos 0 = 1$

$\vec{A} = \hat{i}$
$\vec{B} = \hat{i}$

$\vec{A} \cdot \vec{B} = AB\cos\theta$
$= 1 \times 1 \times \cos 0$
$\vec{A} \cdot \vec{B} = 1$

$\hat{i} \cdot \hat{i} = 1$

Dot product ka kam ka dabba.

* $\vec{A}\cdot\vec{B} = AB\cos\theta$
* $\cos\theta = \frac{\vec{A}\cdot\vec{B}}{AB}$
* $\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$
* $\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{i}\cdot\hat{k} = 0$
* $\vec{A}\perp\vec{B} \Rightarrow \vec{A}\cdot\vec{B} = 0$

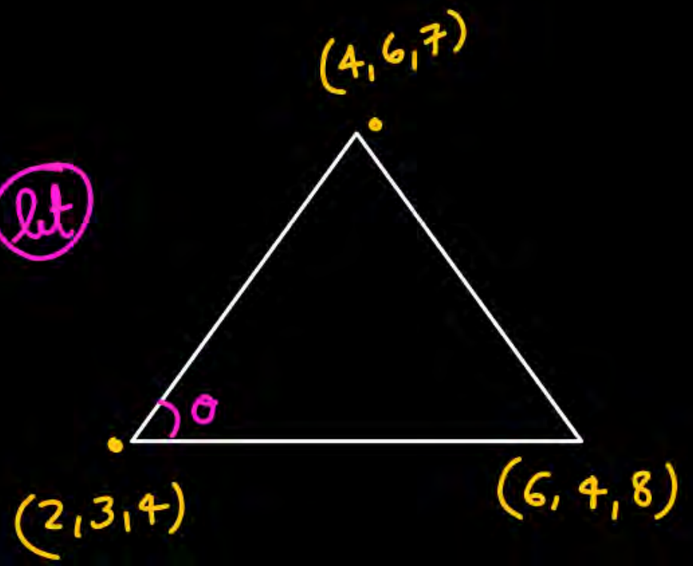
* $\vec{A}\cdot\vec{A} = A^2$
* $\vec{A}\cdot(\vec{B}+\vec{C}) = \vec{A}\cdot\vec{B} + \vec{A}\cdot\vec{C}$
* $(\vec{A}+\vec{B})\cdot(\vec{C}+\vec{D}) = \vec{A}\cdot\vec{C} + \vec{A}\cdot\vec{D} + \vec{B}\cdot\vec{C} + \vec{B}\cdot\vec{D}$

$\vec{A}\cdot\vec{B} = AB\cos\theta$

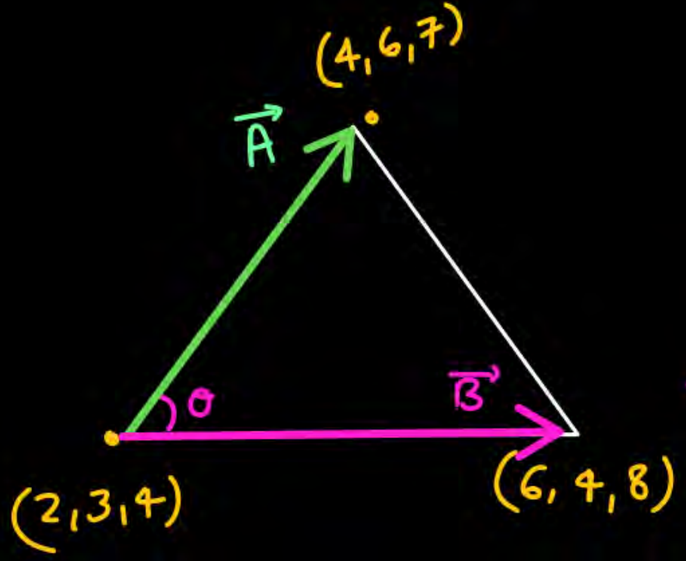
$\vec{A}\cdot\vec{A} = A\cdot A\cos 0 = A^2$

Q

let



$\Rightarrow$

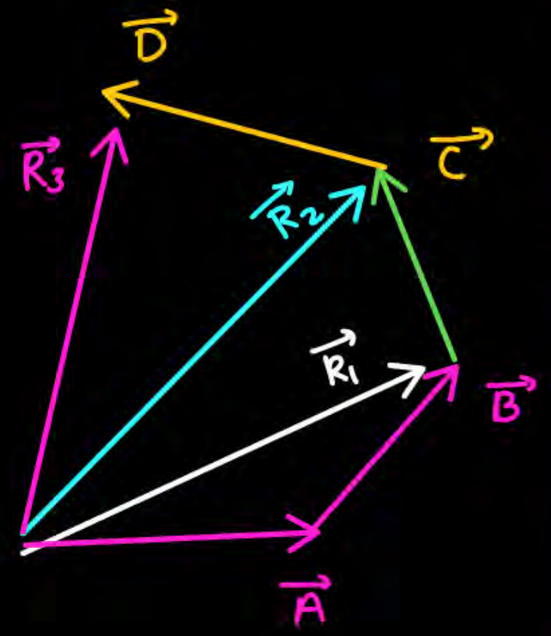


$\vec{A} = 2\hat{i} + 3\hat{j} + 3\hat{k}$

$\vec{B} = 4\hat{i} + \hat{j} + 4\hat{k}$

$\cos\theta = \checkmark$

Polygon Law



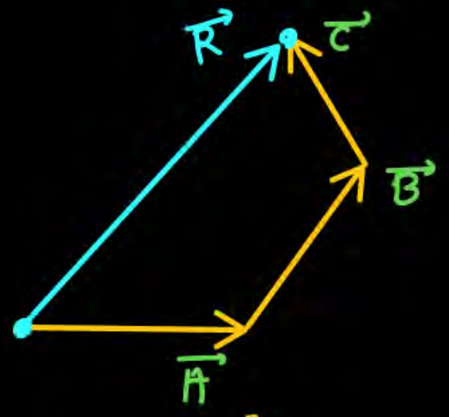
$\vec{R}_1 = \vec{A} + \vec{B}$

$\vec{R}_1 + \vec{C} = \vec{R}_2$

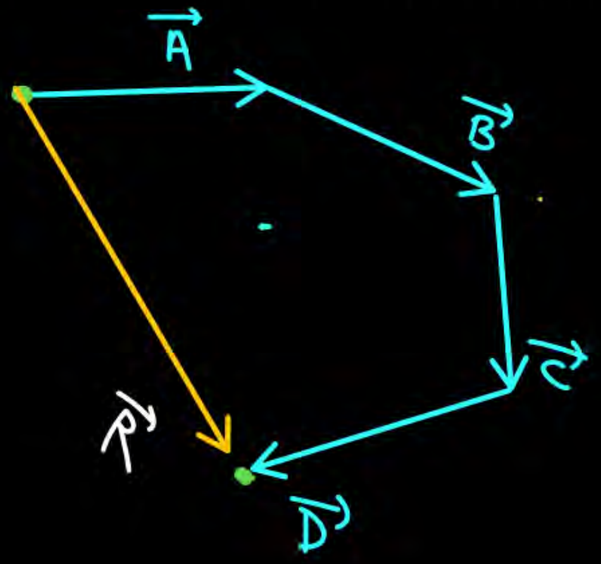
$\Rightarrow \vec{R}_2 = \vec{A} + \vec{B} + \vec{C}$

$\vec{R}_2 + \vec{D} = \vec{R}_3$

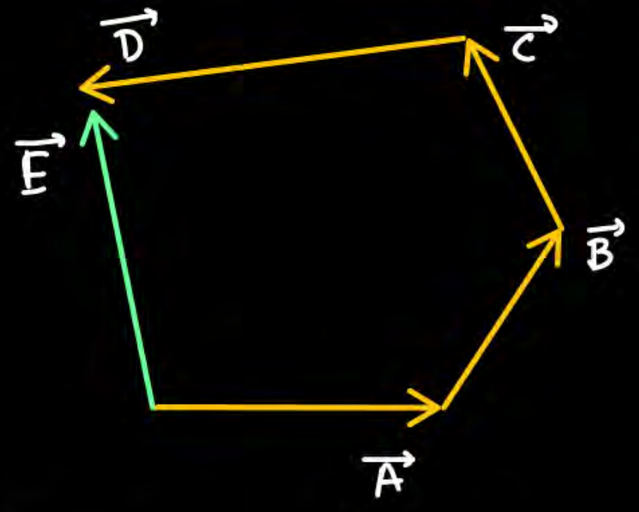
$\vec{R}_3 = \vec{A} + \vec{B} + \vec{C} + \vec{D}$



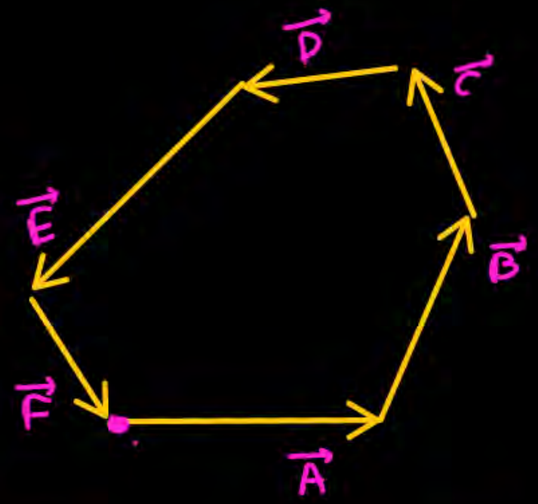
$\vec{A} + \vec{B} + \vec{C} = \vec{R}$



$\vec{A} + \vec{B} + \vec{C} + \vec{D} = \vec{R}$

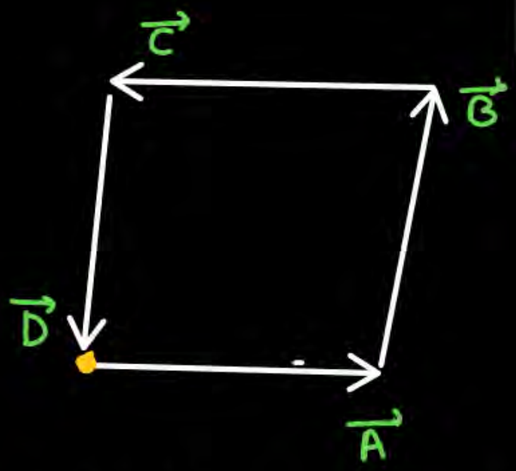


$$\vec{E} = \vec{A} + \vec{B} + \vec{C} + \vec{D}$$



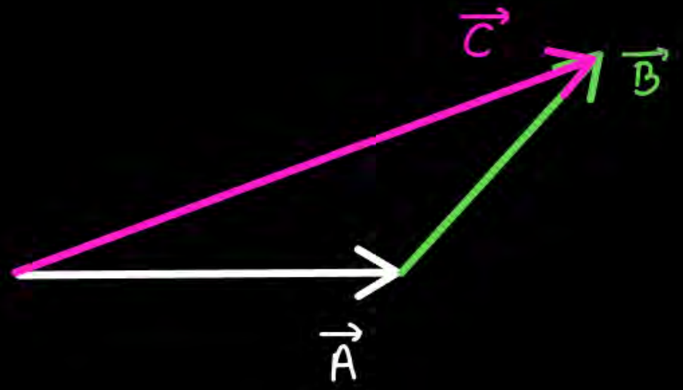
$$\vec{A} + \vec{B} + \vec{C} + \vec{D} + \vec{E} + \vec{F} = \vec{0}$$

Zero Vector
Null Vector
(Arbitrary dir^{n})

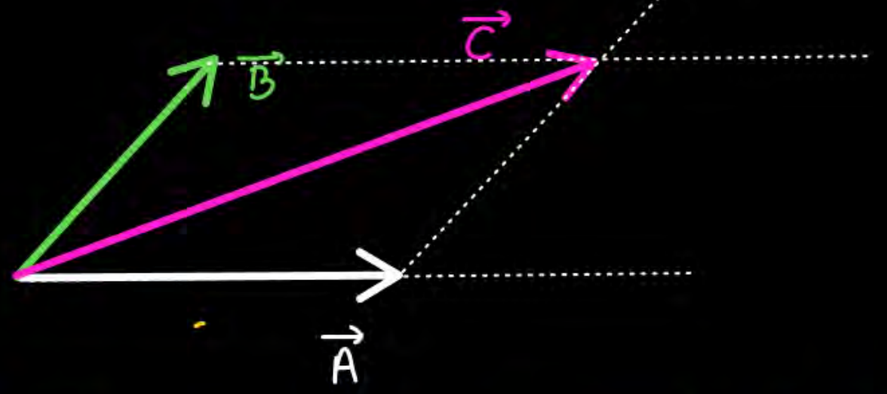


$$\vec{A} + \vec{B} + \vec{C} + \vec{D} = \vec{0}$$

*

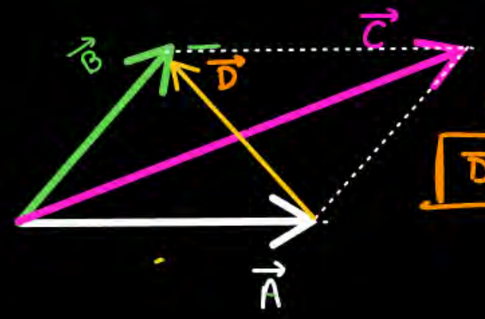


$\vec{A}+\vec{B}=\vec{C}$, (Triangle law)



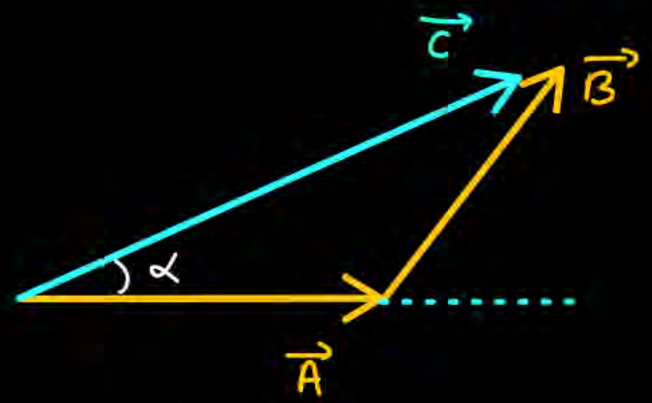
* Parallelogram Law ⇒ Daigonal of the parallelogram made that gives Resultant of $\vec{A}$ & $\vec{B}$.

$\vec{A}+\vec{B}=\vec{C}$ & another daigonal will give difference of $\vec{A}$ & $\vec{B}$.

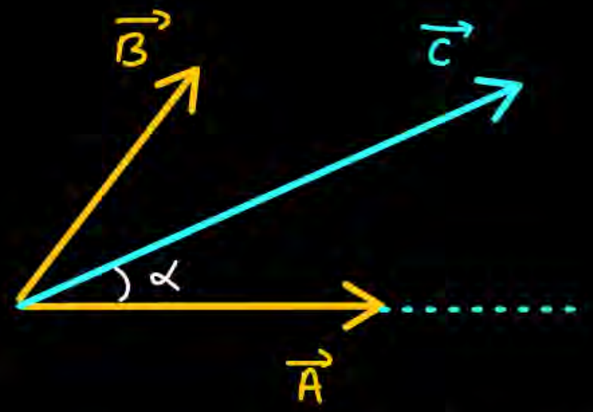


$\vec{A}+\vec{D}=\vec{B}$

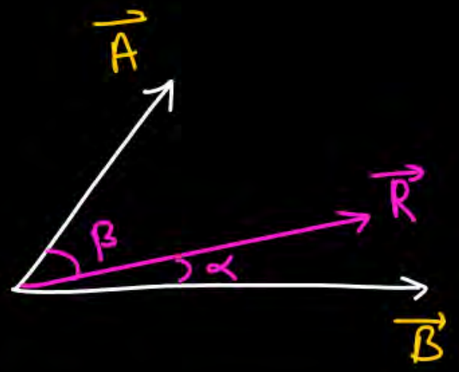
$\boxed{\vec{D}=\vec{B}-\vec{A}}$



$$\tan\alpha = \frac{B\sin\theta}{A+B\cos\theta}$$



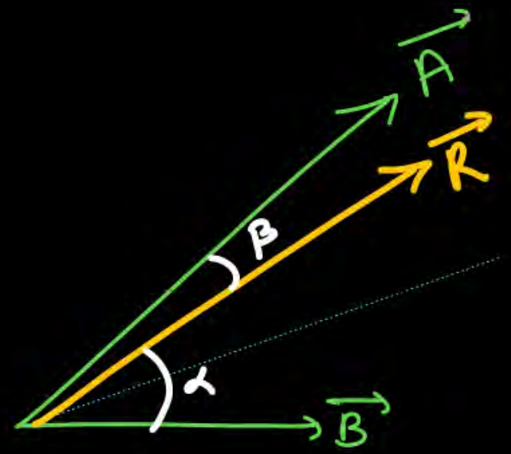
$\vec{A}$ & $\vec{B}$ ka resultant Unke beech me se kahin se (Niklega)
& jo vector Bhari hoga uski taraf jukha hoga.



If

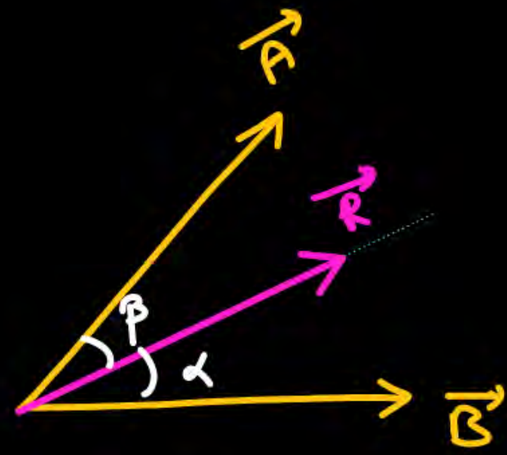
$B > A$

$\alpha < \beta$



* If $A > B$

$\beta < \alpha$



* $A = B$

$\alpha = \beta$

If both vector have equal magnitude then their resultant will be along their angle bisector

Q If magnitude of $\vec{A}$ & $\vec{B}$ are 20N & 19N and angle between them is 40°.

SSSA

If their resultance makes angle α & β with $\vec{B}$ & $\vec{A}$.

which of the statement true.

① $\alpha > \beta$ ✓

② $\alpha < \beta$ ✗

③ $\alpha = \beta$ ✗

④ $\alpha \simeq 39°$ ✗

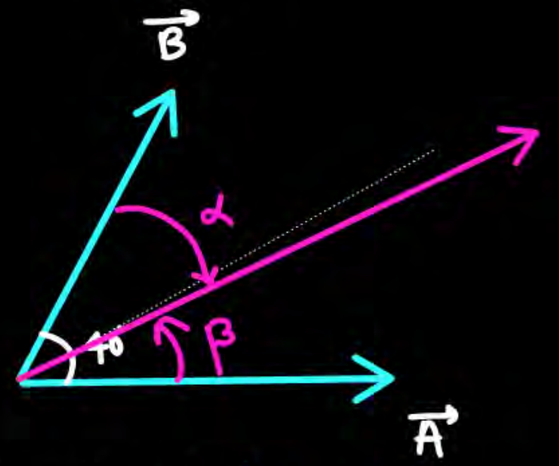
⑤ $\beta \simeq 39°$ ✗

⑥ $\alpha \simeq 19°$ ✗

⑦ $\alpha \simeq 21°$ ✓

⑧ $\beta \simeq 19°$ ✓

⑨ $\beta \simeq 21°$ ✗



* $\alpha > \beta$, (slightly)

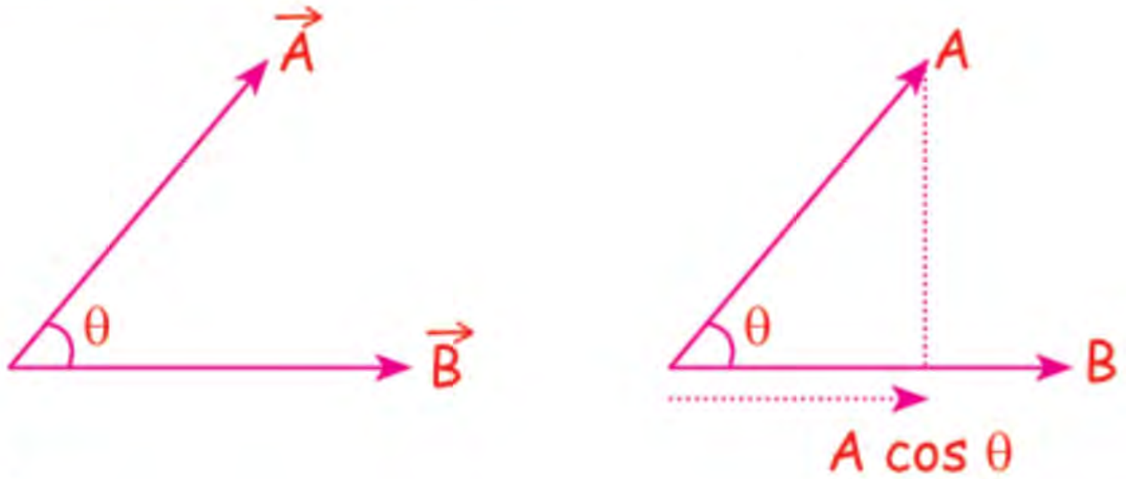
* $\alpha + \beta = 40°$

$\alpha > 20°$, (slightly)

Dot Product

$\vec{A}\cdot\vec{B} = A\,B\,\text{Cos}\theta$

Here, A and B are the magnitude of $\vec{A}$ and $\vec{B}$ and θ is the angel between $\vec{A}$ and $\vec{B}$.



$\vec{A}\cdot\vec{B} = AB\cos\theta$

$\vec{A}\cdot\vec{B} = A(B\cos\theta)$

= A (magnitude of component $\vec{B}$ along $\vec{A}$)

- $\vec{A}\cdot\vec{B} = (A\cos\theta)B$

  = (magnitude of componentof $\vec{A}$ along $\vec{B}$) B
- If $\vec{A}\perp\vec{B} \Rightarrow \vec{A}\cdot\vec{B}$ $A\cdot B\cos 90° = 0$
- $\hat{i}\cdot\hat{j} = 1\times 1\times\cos 90° = 0$
- $\hat{i}\cdot\hat{k} = 1\times 1\times\cos 90° = 0$
- $\hat{j}\cdot\hat{k} = 1\times 1\times\cos 90° = 0$
- $\vec{A}\cdot\vec{B}$ = number ayega scalar.
- $\hat{i}\cdot\hat{i} = 1\times 1\times\cos 0° = 1$
- $\hat{j}\cdot\hat{j} = 1\times 1\times\cos 0° = 1$
- $\hat{k}\cdot\hat{k} = 1\times 1\times\cos 0° = 1$
- $[\vec{A}\cdot\vec{A} = A^2]$

If $\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$

$\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$

then $\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$

$\because \vec{A} \cdot \vec{B} = AB \cos\theta \Rightarrow \cos\theta = \dfrac{\vec{A} \cdot \vec{B}}{AB}$

★ Dot product of two vector is commutative

$\vec{A} . \vec{B} = \vec{B} . \vec{A}$

★ $\vec{A} . \vec{A} = A^2$

★ $(\vec{A} + \vec{B}) \cdot (\vec{A} + \vec{B}) = A^2 + B^2 + \vec{A} . \vec{B} + \vec{B} . \vec{A}$

$= A^2 + B^2 + 2\vec{A} . \vec{B}$

Q. Find dot product of $\vec{A}$ and $\vec{B}$.

$$\vec{A} = 2\hat{i} + 3\hat{j}$$

$$\vec{B} = 4\hat{i} + 5\hat{j}$$

$$\vec{A} \cdot \vec{B} = 8 + 15 = 23$$

Q. $\vec{A} = 3\hat{i} + 4\hat{j} + 5\hat{k}$

$\vec{B} = 2\hat{i} + 3\hat{j} + 6\hat{k}$ Find angle between $\vec{A}$ & $\vec{B}$

Sol. $\vec{A}\cdot\vec{B} = 6 + 12 + 30 = 48,\ A = 5\sqrt{2}\ ,\ B = 7$

$\vec{A}\cdot\vec{B} = AB\cos\theta$

$48 = 5\sqrt{2} \times 7\cos\theta$

$$\frac{48}{35\sqrt{2}} = \cos\theta$$

Q. Find angle between $\vec{A}$ & $\vec{B}$ if

$\vec{A} = \hat{i} + \hat{j} + \hat{k}$, $\vec{B} = \hat{i} - \hat{j} + \hat{k}$

Sol. $\vec{A} \cdot \vec{B} = AB\cos\theta$

$1 = \sqrt{3} \times \sqrt{3}\cos\theta$

$\frac{1}{3} = \cos\theta$

$\theta = \cos^{-1}\left(\frac{1}{3}\right)$

Homework

— KPP.

Thank You